

Stat 344  
Life Contingencies I

Chapter 2: Survival models

# The future lifetime random variable — Notation

- We are interested in analyzing and describing the future lifetime of an individual.
- We use  $(x)$  to denote a life age  $x$  for  $x \geq 0$ .
- Then the *random variable* describing the future lifetime, or time until death, for  $(x)$  is denoted by  $T_x$  for  $T_x \geq 0$ .
- We also have notation for the density, distribution and survival functions of  $T_x$ :

Density:  $f_x(t) = \frac{d}{dt}F_x(t)$

Distribution:  $F_x(t) = Pr[T_x \leq t]$

Survival:  $S_x(t) = Pr[T_x > t] = 1 - F_x(t)$

# The future lifetime random variable — Relationships

We now consider the set of random variables  $\{T_x\}_{x \geq 0}$  and the relationships between them:

Important Assumption

$$Pr[T_x \leq t] = Pr[T_0 \leq x + t | T_0 > x]$$

Important Consequence

$$S_0(x + t) = S_0(x) \cdot S_x(t)$$

This idea can be generalized:

Important General Relationship

$$S_x(t + u) = S_x(t) \cdot S_{x+t}(u)$$

# Survival function conditions and assumptions

We will require any survival function corresponding to a future lifetime random variable to satisfy the following conditions:

## Necessary and Sufficient Conditions

- ①  $S_x(0) = 1$
- ②  $\lim_{t \rightarrow \infty} S_x(t) = 0$
- ③  $S_x(t)$  must be a non-increasing function of  $t$

In addition, we will make the following assumptions for the survival functions used in this course:

## Assumptions

- ①  $S_x(t)$  is differentiable for all  $t > 0$ .
- ②  $\lim_{t \rightarrow \infty} t \cdot S_x(t) = 0$
- ③  $\lim_{t \rightarrow \infty} t^2 \cdot S_x(t) = 0$

# Future lifetime random variable example

Assume that the survival function for a newborn is given by

$$S_0(t) = 8(t + 2)^{-3}$$

- ① Verify that this is a valid survival function.
- ② Find the density function associated with the future lifetime random variable  $T_0$ , that is, find  $f_0(t)$ .  $[24(t + 2)^{-4}]$
- ③ Find the probability that a newborn dies between the ages of 1 and 2.  $[0.171296]$
- ④ Find the survival function corresponding to the random variable  $T_{10}$ .  $[(1 + t/12)^{-3}]$

# Future lifetime random variable example

Assume that the survival function for a newborn is given by

$$S_0(t) = 8(t+2)^{-3}$$

Verify that this is a valid survival function.

$$S_0(0) = 8(2)^{-3} = 1 \quad \checkmark$$

$$\lim_{t \rightarrow \infty} S_0(t) = \lim_{t \rightarrow \infty} 8(t+2)^{-3} = \lim_{t \rightarrow \infty} t^{-3} = 0 \quad \checkmark$$

$$S'_0(t) = -24(t+2)^{-4} < 0 \quad \checkmark$$

# Future lifetime random variable example

Assume that the survival function for a newborn is given by

$$S_0(t) = 8(t + 2)^{-3}$$

Find the density function associated with the future lifetime random variable  $T_0$ , that is, find  $f_0(t)$ .  $[24(t + 2)^{-4}]$

$$\frac{d}{dt} \left( 1 - 8(t+2)^{-3} \right) = 24(t+2)^{-4}$$

# Future lifetime random variable example

Assume that the survival function for a newborn is given by

$$S_0(t) = 8(t + 2)^{-3}$$

Find the probability that a newborn dies between the ages of 1 and 2. [0.171296]

$$S_0(1) - S_0(2)$$

$$F_0(2) - F_0(1)$$



# Future lifetime random variable example

Assume that the survival function for a newborn is given by

$$S_0(t) = 8(t+2)^{-3}$$

Find the survival function corresponding to the random variable  $T_{10}$ .  $[(1 + t/12)^{-3}]$

$$S_0(10+t) = S_0(10) \underbrace{S_{10}(t)}$$

$$\frac{S_0(10+t)}{S_0(10)} = S_{10}(t)$$

$$\frac{8(10+t+2)^{-3}}{8(12)^{-3}} = \left(\frac{12+t}{12}\right)^{-3} = \left(1 + \frac{t}{12}\right)^{-3}$$

# Force of Mortality

- We will define the **force of mortality** as

$$\mu_x = \lim_{dx \rightarrow 0^+} \frac{Pr [T_0 \leq x + dx | T_0 > x]}{dx}$$

- Then for small values of  $dx$ , we can use the approximation

$$\mu_x dx \approx Pr [T_0 \leq x + dx | T_0 > x]$$

- We can relate the force of mortality to the survival function:

$$\mu_x = \frac{-\frac{d}{dx} S_0(x)}{S_0(x)} \quad \text{and} \quad S_x(t) = \exp \left\{ - \int_0^t \mu_{x+s} ds \right\}$$

- An alternative relationship that's sometimes more convenient:

$$\mu_x = -\frac{d}{dx} \ln S_0(x)$$

# Some commonly used forms for the force of mortality

It is sometimes convenient to describe the future lifetime random variable by explicitly modeling the force of mortality. A couple of the parametric forms that have been used are:

- Gompertz:  $\mu_x = Bc^x$ ,  $0 < B < 1$ ,  $c > 0$
- Makeham:  $\mu_x = A + Bc^x$ ,  $0 < B < 1$ ,  $c > 0$

If there is a **limiting age**, it is typically denoted by  $\omega$ .

# Force of mortality example

Again, assume that the survival function for a newborn is given by

$$S_0(x) = 8(x + 2)^{-3}$$
$$\frac{\frac{d}{dt} S_0(t)}{S_0(t)}$$

- ① Find the force of mortality,  $\mu_x$ .  $[3/(\textcolor{blue}{x} + 2)]$
- ② Consider the shape of  $\mu_x$  as a function of  $x$ .
  - Does it seem to be a realistic survival model for describing human lifetimes?
  - What would you expect the force of mortality corresponding to humans to look like?

# Actuarial Notation

Thus far we have described the properties of the future lifetime random variable using statistical notation. Actuaries have also developed their own (different) notation to describe many of the same concepts:

- ${}_t p_x = \Pr [T_x > t] = S_x(t)$
- ${}_t q_x = \Pr [T_x \leq t] = F_x(t)$
- ${}_u|{}_t q_x = \Pr [u < T_x \leq u + t] = F_x(u + t) - F_x(u)$

In any of the above expressions, the subscript  $t$  may be omitted when its value is 1.

The density of the random variable  $T_x$  can be written as  $f_X(t) = {}_t p_x \mu_{x+t}$ .

# Some Basic Relationships

We can express some relationships in this actuarial notation:

- ${}_t p_x = 1 - {}_t q_x$

- ${}_u | {}_t q_x = {}_u p_x - {}_{u+t} p_x$

- ${}_{t+u} p_x = {}_t p_x {}_u p_{x+t}$

- ${}_u | {}_t q_x = {}_u p_x {}_t q_{x+u}$

- ${}_t p_x = \exp \left\{ - \int_0^t \mu_{x+s} ds \right\}$

- ${}_t q_x = \int_0^t {}_s p_x \mu_{x+s} ds$

# Examples

- ①  ${}_5p_{30} = 0.9$ ,  ${}_5p_{35} = 0.8$ . Find  ${}_{10}p_{30}$ . [0.72]

$${}_{10}P_{30} = {}_5P_{30} \cdot {}_5P_{35}$$

- ②  ${}_3p_{80} = 0.6$ ,  ${}_2p_{81} = 0.7$ . Find  $p_{80}$ . [0.857]

$${}_3P_{80} = P_{80} \cdot {}_2P_{81}$$

- ③  ${}_2|q_{60} = 0.216$ ,  ${}_2p_{61} = 0.56$ ,  $p_{62} = 0.7$ . Find  $q_{60}$ . [0.1]

$${}_2P_{61} = P_{61} P_{62} \quad P_{61} = \frac{0.56}{0.7} \quad {}_2|q_{60} = P_{60} P_{61} q_{62}$$

- ④ Show that  $\frac{d}{dt} {}_t p_x = -{}_t p_x \mu_{x+t}$ .

$$\mu_{x+t} = \frac{-\frac{d}{dt} {}_t P_x}{{}_t P_x}$$

# Mean of the Future Lifetime Random Variable

We can consider various properties of the future lifetime RV:

- One quantity that is often of interest is the mean of the future lifetime random variable,  $E[T_x]$ .
- The actuarial symbol for this mean is  $\overset{\circ}{e}_x$ :

$$\begin{aligned}\overset{\circ}{e}_x &= \int_0^{\infty} \underbrace{t f_x(t)} dt \\ &= \int_0^{\infty} \underbrace{t {}_t p_x \mu_{x+t}} dt \\ &= \int_0^{\infty} {}_t p_x dt\end{aligned}$$

- A related quantity is  $\overset{\circ}{e}_{x:\overline{n}|} = \int_0^n {}_t p_x dt$



# Variance of the Future Lifetime Random Variable

In order to calculate the variance of  $T_x$ , we need its second moment:

$$\begin{aligned} E [T_x^2] &= \int_0^\infty t^2 f_x(t) dt \\ &= \int_0^\infty t^2 {}_t p_x \mu_{x+t} dt \\ &= 2 \int_0^\infty t {}_t p_x dt \end{aligned}$$

Then we can calculate the variance in the usual way:

$$\begin{aligned} V [T_x] &= E [T_x^2] - (E [T_x])^2 \\ &= E [T_x^2] - \left( {}^\circ e_x \right)^2 \end{aligned}$$

# Example

You are given the following survival function for a newborn:

$$S_0(t) = \frac{1}{10} \sqrt{100 - t}, \quad 0 \leq t \leq 100$$

- ① Find  ${}^{\circ}e_{19}$ . [54]
- ② Calculate  $Var(T_{19})$ . [583.2]
- ③ Find the mode of  $T_{19}$ . [81]
- ④ Verify that  $S_0(t)$  is a valid survival function.

# Example

You are given the following survival function for a newborn:

$$S_0(t) = \frac{1}{10} \sqrt{100 - t}, \quad 0 \leq t \leq 100$$

Find  ${}^{\circ}e_{19}$ . [54]

$${}_tP_{19} = \frac{S_0(19+t)}{S_0(19)}$$

$${}^{\circ}e_{19} = \int_0^{81} {}_tP_{19} dt = \int_0^{81} \frac{\sqrt{81-t}}{9} dt = 54$$

# Example

You are given the following survival function for a newborn:

$$S_0(t) = \frac{1}{10} \sqrt{100 - t}, \quad 0 \leq t \leq 100$$

Calculate  $\text{Var}(T_{19})$ . [583.2]

$$E(T_{19}^2) = 2 \int_0^{81} t {}_tP_{19} = 2 \int_0^{81} \frac{t \sqrt{81-t}}{9} dt$$
$$3499.2$$

$$\text{Var} = 3499.2 - 54^2 = 583.2$$

# Example

You are given the following survival function for a newborn:

$$S_0(t) = \frac{1}{10} \sqrt{100 - t}, \quad 0 \leq t \leq 100$$

Find the mode of  $T_{19}$ . [81]

$$f(t) = -\frac{d}{dt} S_{19}(t)$$

$$\frac{d}{dt} f(t)$$

# Example

You are given the following survival function for a newborn:

$$S_0(t) = \frac{1}{10} \sqrt{100 - t}, \quad 0 \leq t \leq 100$$

Verify that  $S_0(t)$  is a valid survival function.

$$S_0(0) = 1$$

$$S_0(100) = 0$$

$$\frac{d}{dt} S_0(t) < 0$$

$10 P_{25}$

$10 g_{45}$

$15/10 g_{45}$

$^0 e_{45}$

$\mu_{45}$

# Summary of notation for the future lifetime RV

We can summarize the statistical and actuarial notation for the properties of  $T_x$  covered thus far:

Concept	Statistical Notation	Actuarial Notation
Expected Value	$E[T_x]$	${}^{\circ}e_x$
Distribution Function	$F_x(t)$	${}_tq_x$
Survival Function	$S_x(t)$	${}_tp_x$
Force of Mortality	$\lambda(x)$ or $\lambda_x$	$\mu_x$
Density	$f_x(t)$	${}_tp_x \mu_{x+t}$
Deferred Mortality Prob.	$Pr[u < T_x \leq u + t]$	${}_u {}_tq_x$



# Curtate Future Lifetime Random Variable

We are often interested in the *integral* number of years lived in the future by an individual. This *discrete* random variable is called the **curtate future lifetime** and is denoted by  $K_x$ .

We can consider the pmf (probability mass function) of  $K_x$ :

$$\begin{aligned}Pr [K_x = k] &= Pr [k \leq T_x < k + 1] \\&= {}_k|q_x \\&= {}_kp_x q_{x+k}\end{aligned}$$

We also have the relation  $K_x = \lfloor T_x \rfloor$ .

# Moments of the Curtate Future Lifetime Random Variable

We can find the mean (denoted by  $e_x$ ) and variance of  $K_x$ :

$$E[K_x] = e_x = \sum_{k=1}^{\infty} k p_x$$

$$E[K_x^2] = 2 \left( \sum_{k=1}^{\infty} k p_x \right) - e_x$$

$$V[K_x] = E[K_x^2] - (e_x)^2$$

There's a term version of this as well:  $e_{x:\overline{n}|} = \sum_{k=1}^n k p_x$

Using the trapezoidal rule, we can also find an approximate relation between the means of  $T_x$  and  $K_x$ :

$$e_x^{\circ} \approx e_x + \frac{1}{2}$$

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

- ① Calculate  $Pr(K_{90} = 1)$ . [0.18]
- ② Calculate  $e_{90}$ . [2.124]
- ③ Calculate  $Var(K_{90})$ . [1.069]
- ④ Calculate  $e_{90:\overline{2}|}$ . [1.62]
- ⑤ Write the pmf of  $K_{90}$ .
- ⑥ Repeat 1 – 5, using the same mortality information, but substituting 91 for 90. [0.24; 1.36; 0.6304; 1.36]

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $Pr(K_{90} = 1)$ . [0.18]

$$p_{90} q_{91} = 0.9 (0.2)$$

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $e_{90}$ . [2.124]

$k$	$P(k)$
0	0.1
1	0.18
2	$0.9(0.8)(0.3) = 0.216$
3	$0.9(0.8)(0.7) = .504$

$${}_1P_{90} = 0.9$$

$${}_2P_{90} = 0.9(0.8)$$

$${}_3P_{90} = 0.9(0.8)(0.7)$$

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $\text{Var}(K_{90})$ . [1.069]

$$1^2 (0.1) + 2^2 (0.216) + 3^2 (0.504) = E(K^2)$$

$$E(K^2) - (E(K))^2 = \text{Var}(K)$$

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $e_{90:\overline{2}|}$ . [1.62]

$k$	$P(k)$
0	0.1
1	0.18
2	$0.9(0.8)(0.3) = 0.216$
<del>2</del> 2	$0.9(0.8)(0.7) = .504$

$$0.1(0) + 1(0.18) + 2(0.216 + .504)$$

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Write the pmf of  $K_{90}$ .



# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $Pr(K_{91} = 1)$ . [0.18]

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $e_{91}$ . [2.124]

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $\text{Var}(K_{91})$ . [1.069]

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Calculate  $e_{91:\overline{2}|}$ . [1.62]

# Example

You are given the following mortality information:

$$q_{90} = 0.1, \quad q_{91} = 0.2, \quad q_{92} = 0.3, \quad q_{93} = 1$$

Write the pmf of  $K_{91}$ .

# $m^{th}$ ly future lifetime random variable

We define the *discrete* random variable  $K_x^{(m)}$  to be the future lifetime, rounded down to the lower  $1/m$  of a year. Some examples:

$T_x$	$K_x$	$K_x^{(2)}$	$K_x^{(4)}$	$K_x^{(12)}$
25.7	25	$25\frac{1}{2}$	$25\frac{2}{4}$	$25\frac{8}{12}$
25.1	25	25	25	$25\frac{1}{12}$

Note: In this definition,  $m$  must be an integer greater than 1.

The pmf of  $K_x^{(m)}$  is similar to that of  $K_x$ :

$$Pr \left[ K_x^{(m)} = k \right] = Pr \left[ k \leq T_x < k + \frac{1}{m} \right] = k|_{\frac{1}{m}} q_x \quad k = 0, \frac{1}{m}, \frac{2}{m}, \dots$$

## Exercise 2.2

$$S_0(x) = \frac{18000 - 110x - x^2}{18000}$$

- ① What is the implied limiting age? [90]
- ② Show that it is a valid survival function.
- ③ Calculate  ${}_{20}p_0$ . [0.8556]
- ④ Derive  $S_{20}(t)$ .  $\left[ \frac{15400 - 150t - t^2}{15400} \right]$
- ⑤ Calculate  ${}_{10|10}q_{20}$ . [0.1169]
- ⑥ Calculate  $\mu_{50}$ . [0.021]

## Exercise 2.2

$$S_0(x) = \frac{18000 - 110x - x^2}{18000}$$

What is the implied limiting age? [90]

$$0 = 18000 - 110x - x^2$$



## Exercise 2.2

$$S_0(x) = \frac{18000 - 110x - x^2}{18000}$$

Show that it is a valid survival function.

$$S_0(0) = \frac{18000}{18000} = 1$$

$$S_0(90)$$

## Exercise 2.2

$$S_0(x) = \frac{18000 - 110x - x^2}{18000}$$

Calculate  ${}_{20}p_0$ . [0.8556]

$$S_0(20)$$

## Exercise 2.2

$$S_0(x) = \frac{18000 - 110x - x^2}{18000}$$

Derive  $S_{20}(t)$ .  $\left[ \frac{15400 - 150t - t^2}{15400} \right]$

$$S_0(20+t) = S_0(20) S_{20}(t)$$

## Exercise 2.2

$$S_0(x) = \frac{18000 - 110x - x^2}{18000}$$

Calculate  ${}_{10|10}q_{20}$ . [0.1169]

$$S_{20}(10) - S_{20}(20)$$

$$\frac{S_0(30) - S_0(40)}{S_0(20)}$$

$$S_{20}(t) = \frac{S_0(20+t)}{S_0(20)}$$

## Exercise 2.2

$$S_0(x) = \frac{18000 - 110x - x^2}{18000}$$

Calculate  $\mu_{50}$ . [0.021]

$$\frac{-\frac{d}{dt} S_0(t)}{S_0(t)}$$

## Examples 2.1, 2.2, and 2.6

$$F_0(t) = \begin{cases} 1 - (1 - \frac{t}{120})^{1/6} & \text{for } 0 \leq t \leq 120 \\ 1 & \text{for } t \geq 120 \end{cases}$$

- ① Calculate the probability that a newborn survives to age 30.  $[0.9532]$
- ② Calculate the probability that (30) dies before age 50.  $[0.0410]$
- ③ Calculate the probability that (40) survives to age 65.  $[0.9395]$
- ④ Derive  $\mu_x$ .  $[1/(120-6x)]$
- ⑤ Calculate  $\overset{\circ}{e}_x$  and  $V(T_x)$  for (30) and (80).  $[77.143, 34.286, 21.396^2, 9.509^2]$

## Examples 2.1, 2.2, and 2.6

$$F_0(t) = \begin{cases} 1 - (1 - \frac{t}{120})^{1/6} & \text{for } 0 \leq t \leq 120 \\ 1 & \text{for } t \geq 120 \end{cases}$$

Calculate the probability that a newborn survives to age 30.

$$S_0(30) = 1 - F_0(30) = 0.9532$$

## Examples 2.1, 2.2, and 2.6

$$F_0(t) = \begin{cases} 1 - (1 - \frac{t}{120})^{1/6} & \text{for } 0 \leq t \leq 120 \\ 1 & \text{for } t \geq 120 \end{cases}$$

Calculate the probability that (30) dies before age 50.

$$\frac{F_0(50) - F_0(30)}{1 - F_0(30)} = \frac{S_0(30) - S_0(50)}{S_0(30)}$$



## Examples 2.1, 2.2, and 2.6

$$F_0(t) = \begin{cases} 1 - (1 - \frac{t}{120})^{1/6} & \text{for } 0 \leq t \leq 120 \\ 1 & \text{for } t \geq 120 \end{cases}$$

Calculate the probability that (40) survives to age 65.

$$S_0(65) = S_0(40) S_{40}(25)$$

$$\frac{S_0(65)}{S_0(40)} = S_{40}(25)$$

# Examples 2.1, 2.2, and 2.6

$$S_0(x) = \left(1 - \frac{x}{120}\right)^{1/6}$$

$$F_0(t) = \begin{cases} 1 - \left(1 - \frac{t}{120}\right)^{1/6} & \text{for } 0 \leq t \leq 120 \\ 1 & \text{for } t \geq 120 \end{cases}$$

Derive  $\mu_x$ .

$$\frac{-\frac{d}{dx} S_0(x)}{S_0(x)}$$

$$\frac{-\frac{1}{6} \left(1 - \frac{x}{120}\right)^{-5/6} \left(-\frac{1}{120}\right)}{\left(1 - \frac{x}{120}\right)^{1/6}}$$

$$\frac{1}{720} \left( \frac{1}{\left(1 - \frac{x}{120}\right)} \right)$$

$$\frac{1}{720 - 6x}$$

# Examples 2.1, 2.2, and 2.6

$${}_tP_x = \frac{S_0(x+t)}{S_0(x)} = \left(1 - \frac{t}{120-x}\right)^{1/6}$$

$$F_0(t) = \begin{cases} 1 - \left(1 - \frac{t}{120}\right)^{1/6} & \text{for } 0 \leq t \leq 120 \\ 1 & \text{for } t \geq 120 \end{cases}$$

Calculate  $\overset{\circ}{e}_x$  and  $V(T_x)$  for (30) and (80). [77.143, 34.286, 21.396<sup>2</sup>, 9.509<sup>2</sup>]

$$\overset{\circ}{e}_x = \int_0^{120-x} \left(1 - \frac{t}{120-x}\right)^{1/6} dt$$

$$y = 1 - \frac{t}{120-x}$$

$$(120-x)(1-y) = t$$

$$\overset{\circ}{e}_x = (120-x) \int_0^1 y^{1/6} dy = \frac{6}{7} (120-x)$$

$$E(T_x^2) = 2 \int_0^{120-x} t {}_tP_x = 2 \int_0^{120-x} t \left(1 - \frac{t}{120-x}\right)^{1/6} dt$$

$$2(120-x)^2 \int_0^1 (y^{1/6} - y^{7/6}) dy = 2(120-x)^2 \left(\frac{6}{7} - \frac{6}{13}\right)$$

## Exercise 2.5

$$F_0(t) = 1 - e^{-\lambda t} \quad \lambda > 0$$

- ① Derive  $S_x(t)$ .  $[e^{-\lambda t}]$
- ② Derive  $\mu_x$ .  $[\lambda]$
- ③ Derive  $e_x$ .  $\left[\frac{1}{e^\lambda - 1}\right]$
- ④ Is this acceptable for human mortality? Why or why not?

## Exercise 2.5

$$F_0(t) = 1 - e^{-\lambda t} \quad \lambda > 0$$

Derive  $S_x(t)$ .  $[e^{-\lambda t}]$

## Exercise 2.5

$$F_0(t) = 1 - e^{-\lambda t} \quad \lambda > 0$$

Derive  $\mu_x$ .  $[\lambda]$

## Exercise 2.5

$$F_0(t) = 1 - e^{-\lambda t} \quad \lambda > 0$$

Derive  $e_x \cdot \left[ \frac{1}{e^\lambda - 1} \right]$

## Exercise 2.5

$$F_0(t) = 1 - e^{-\lambda t} \quad \lambda > 0$$

Is this acceptable for human mortality? Why or why not?



## Exercise 2.6

$$p_x = 0.99$$

$$p_{x+1} = 0.985$$

$${}_3p_{x+1} = 0.95$$

$$q_{x+3} = 0.02$$

Calculate

①  $p_{x+3}$  [0.98]

②  ${}_2p_x$  [0.97515]

③  ${}_2p_{x+1}$  [0.96939]

④  ${}_3p_x$  [0.95969]

⑤  ${}_{1|2}q_x$  [0.03031]